

EXERCISES [MAI 2.10]

ASYMPTOTES

SOLUTIONS

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A. Paper 1 questions (SHORT)

1.

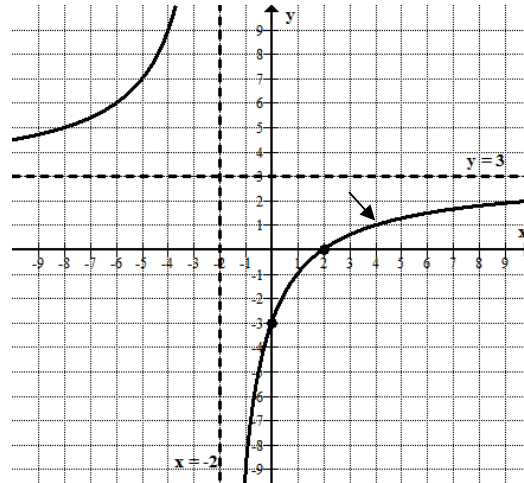
Function	Vertical asymptote	Horizontal asymptote
$y = \frac{2x+7}{x-5}$	$x = 5$	$y = 2$
$y = \frac{2x+7}{x-5} + 1$	$x = 5$	$y = 3$
$y = \frac{2}{x-5}$	$x = 5$	$y = 0$
$y = \frac{2}{x-5} + 1$	$x = 5$	$y = 1$
$y = \frac{2}{x-5} - 1$	$x = 5$	$y = -1$
$y = \frac{x+7}{2x-3}$	$x = \frac{3}{2}$	$y = \frac{1}{2}$
$y = \frac{-4x+1}{2x-6}$	$x = 3$	$y = -2$
$y = \frac{-4x+1}{2x-6} + 3$	$x = 3$	$y = 1$
$y = \frac{5-x}{5+x}$	$x = -5$	$y = -1$

2.

Function	V. A.	H. A.
$y = e^x$	---	$y = 0$
$y = e^{2x}$	---	$y = 0$
$y = 3e^{2x}$	---	$y = 0$
$y = 3e^{2x} + 5$	---	$y = 5$
$y = 3e^{2x} - 5$	---	$y = -5$
$y = 3e^{-2x}$	---	$y = 0$
$y = 3e^{-2x} - 1$	---	$y = -1$
$y = 3e^{-2x} + 2$	---	$y = 2$
$y = 2^x$	---	$y = 0$
$y = 2^x + 3$	---	$y = 3$
$y = 2^{-x} + 3$	---	$y = 3$

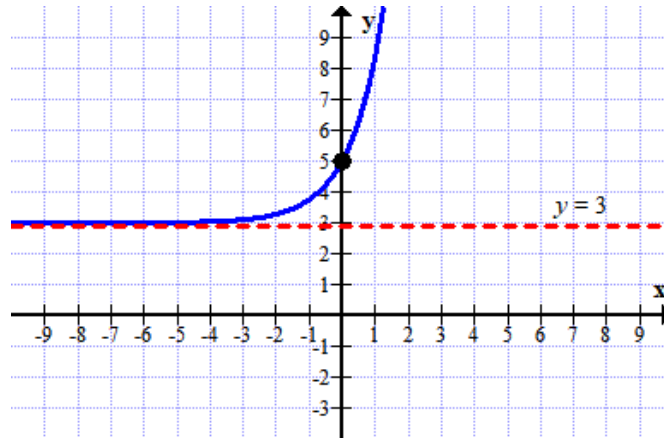
Function	V. A.	H. A.
$y = \ln x$	$x = 0$	---
$y = 2 \ln x$	$x = 0$	---
$y = 2 \ln 3x$	$x = 0$	---
$y = \ln(x-3)$	$x = 3$	---
$y = \log x$	$x = 0$	---
$y = \log(x-3)$	$x = 3$	---
$y = \log(x-3) + 2$	$x = 3$	---

3. (a) H.A.: $y=3$, V.A.: $x=-2$
 (b) x -intercept: $x=2$, y -intercept: $y=-3$
 (c) $\frac{3x-6}{x+2} = 1 \Leftrightarrow 3x-6 = x+2 \Leftrightarrow x=4$ so $(4,1)$ (or by GDC)
 (d)

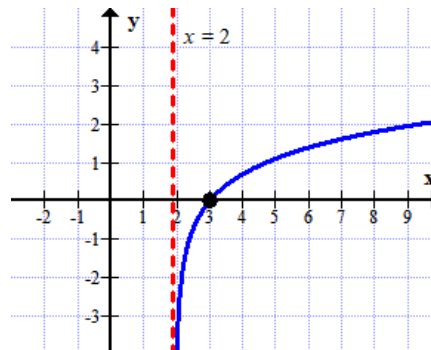


4. (a) $a=3$, $b=-6$, $c=2$
 (b) $f(4)=1$, $f^{-1}(1)=4$.
 (c) $-2 < x \leq 4$

5.



6.



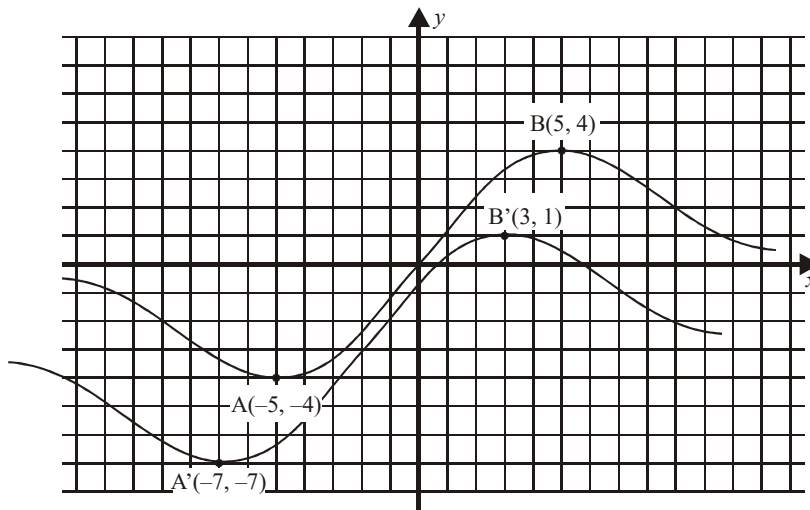
7. (a)

$y = f(x) + 2$	$y = 3$	$y = f(x + 2)$	$y = 1$
$y = f(x) - 2$	$y = -1$	$y = f(x - 2)$	$y = 1$
$y = 2f(x)$	$y = 2$	$y = f(2x)$	$y = 1$
$y = f(x)/2$	$y = 1/2$	$y = f(x/2)$	$y = 1$
$y = -f(x)$	$y = -1$	$y = f(-x)$	$y = 1$

(b)

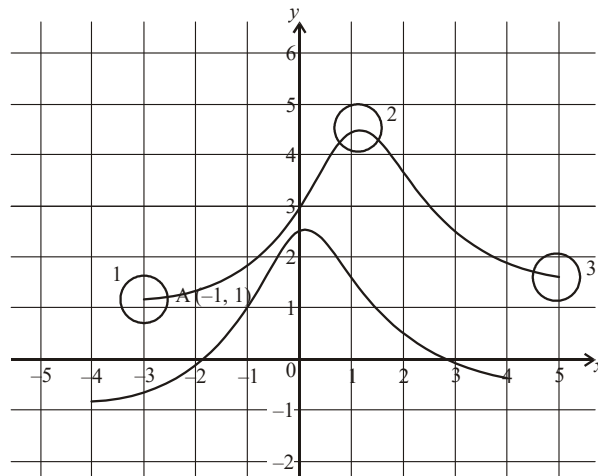
$y = f(x) + 2$	$x = 2$	$y = f(x + 2)$	$x = 0$
$y = f(x) - 2$	$x = 2$	$y = f(x - 2)$	$x = 4$
$y = 2f(x)$	$x = 2$	$y = f(2x)$	$x = 1$
$y = f(x)/2$	$x = 2$	$y = f(x/2)$	$x = 4$
$y = -f(x)$	$x = 2$	$y = f(-x)$	$x = -2$

8. (a)



(b) Asymptote: $y = -3$

9. (a)



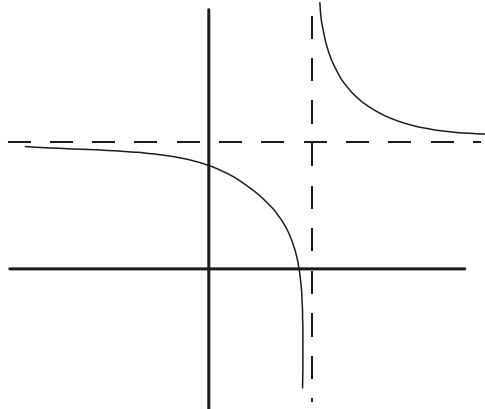
(b) $y = 1$ (must be an equation)

(c) (0, 3)

10. (a) (i) $p = 2$ (ii) $10 = \frac{q}{3-2}$ so $q = 10$

(b) Reflection, in x -axis

11. (a)



(b) (i) $y = 3, x = \frac{5}{2}$ (must be equations)

(ii) $x = \frac{7}{3}$ ($\cong 2.33$)

(iii) $y = \frac{14}{5}$ ($= 2.8$)

B. Paper 2 questions (LONG)

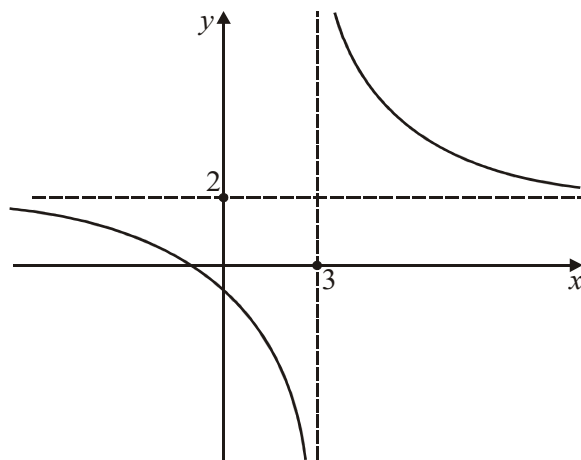
12. (a) (i) H. Asymptote at: $y = 2$ V. Asymptote at $x = 3$

(ii) $P(3, 2)$

(b) $y = 0 \Rightarrow x = -\frac{1}{2}$

$x = 0 \Rightarrow y = -\frac{1}{3}$

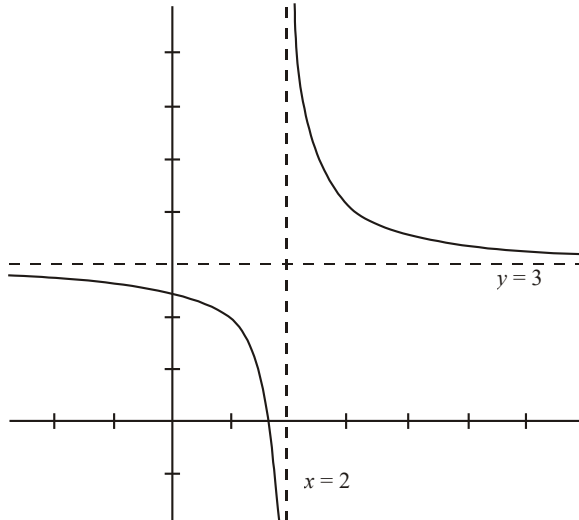
(c)



(d) Range of f : $y \neq 2$

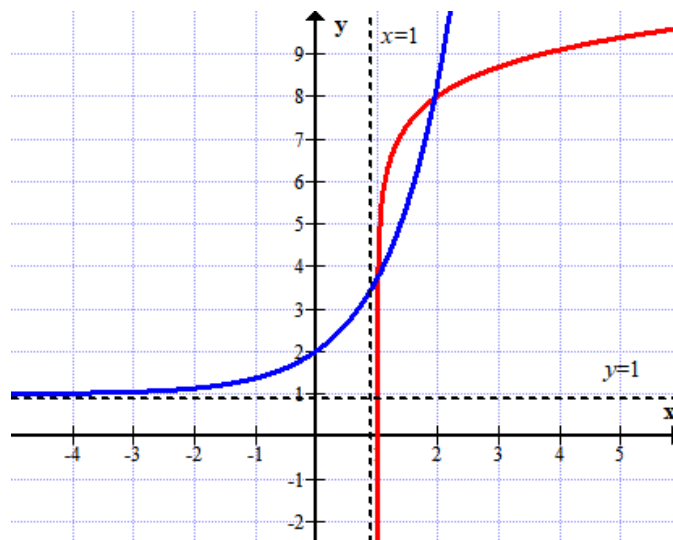
(e) Domain of f^{-1} : $x \neq 2$ Range of f^{-1} : $y \neq 3$

13. (a) Solve for x in terms of y $x = \frac{y+5}{3}$ $f^{-1}(x) = \frac{x+5}{3}$
- (b) $(g^{-1} \circ f)(x) = (3x-5)+2 = 3x-3$
- (c) $\frac{x+3}{3} = 3x-3$ ($x+3 = 9x-9$) so $x = \frac{12}{8}$ (or by GDC $x = 1.5$)
- (d) (i)



- (ii) (Vertical asymptote) $x = 2$,
(Horizontal asymptote) $y = 3$

14. (a) $y = 1$
- (b) $x = 1$
- (c) For f , Domain: $x \in R$ Range: $y > 1$
For g , Domain: $x > 1$ Range: $y \in R$
- (d)



- (e) Points of intersection at $x = 1.01$ and $x = 1.94$
Hence $1.01 \leq x \leq 1.94$